Problem 52

Assuming the $+x$-axis is horizontal to the right for the vectors given in the following figure, use the analytical method to find the following resultants: (a) $11.67\hat{\mathbf{i}} + 8.99\hat{\mathbf{j}}$, (b) $9.01\hat{\mathbf{i}} - 6.40\hat{\mathbf{j}}$, (c) $\overrightarrow{\mathbf{D}} + \overrightarrow{\mathbf{F}}$, (d) $5.65\hat{\mathbf{i}} + 1.01\hat{\mathbf{j}}$, (e) $\overrightarrow{\mathbf{D}} - \overrightarrow{\mathbf{F}}$, (f) $\overrightarrow{\mathbf{A}} + 2\overrightarrow{\mathbf{F}}$, (g) $\overrightarrow{\mathbf{C}} - 2\overrightarrow{\mathbf{D}} + 3\overrightarrow{\mathbf{F}}$, and (h) $\overrightarrow{\mathbf{A}} - 4\overrightarrow{\mathbf{D}} + 2\overrightarrow{\mathbf{F}}$.

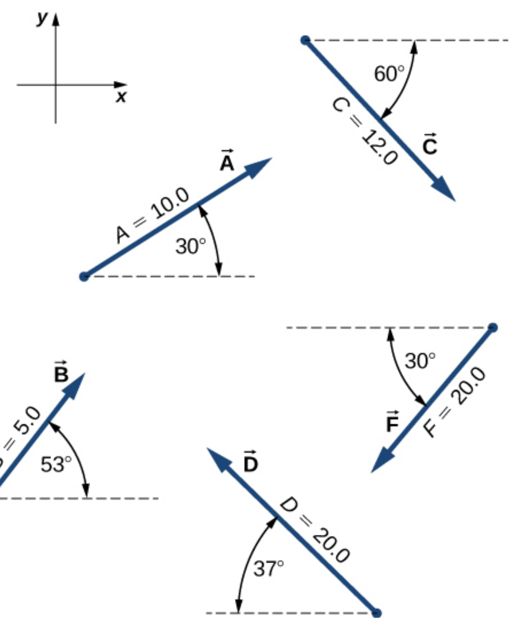

Figure 2.33

Solution

Part (a)

The resultant is given as $\overrightarrow{\mathbf{R}} = 11.67\hat{\mathbf{i}} + 8.99\hat{\mathbf{j}}$. Its magnitude is

$$R = |\overrightarrow{\mathbf{R}}| = \sqrt{(11.67)^2 + (8.99)^2} \approx 14.7,$$

and the direction is

$$\theta = \tan^{-1}\left(\frac{8.99}{11.67}\right) \approx 37.6°,$$

or 37.6° counterclockwise from the positive x-axis.

Part (b)

The resultant is given as $\overrightarrow{\mathbf{R}} = 9.01\hat{\mathbf{i}} - 6.40\hat{\mathbf{j}}$. Its magnitude is

$$R = |\overrightarrow{\mathbf{R}}| = \sqrt{(9.01)^2 + (-6.40)^2} \approx 11.1,$$

and the direction is

$$\theta = \tan^{-1}\left(\frac{-6.40}{9.01}\right) \approx -35.4°,$$

or 35.4° clockwise from the positive x-axis.

For the remaining parts, note that when an angle above or below a horizontal is given, multiplying the magnitude by the cosine of the angle gives the horizontal component, and multiplying the magnitude by the sine of the angle gives the vertical component.

Part (c)

The resultant is

$$\begin{aligned}\vec{\mathbf{D}} + \vec{\mathbf{F}} &= (-20.0\cos 37^\circ\,\hat{\mathbf{i}} + 20.0\sin 37^\circ\,\hat{\mathbf{j}}) + (-20.0\cos 30^\circ\,\hat{\mathbf{i}} - 20.0\sin 30^\circ\,\hat{\mathbf{j}}) \\ &= (-20.0\cos 37^\circ - 20.0\cos 30^\circ)\hat{\mathbf{i}} + (20.0\sin 37^\circ - 20.0\sin 30^\circ)\hat{\mathbf{j}} \\ &\approx -33.3\hat{\mathbf{i}} + 2.04\hat{\mathbf{j}}.\end{aligned}$$

Its magnitude is

$$\left|\vec{\mathbf{D}} + \vec{\mathbf{F}}\right| = \sqrt{(-33.3)^2 + (2.04)^2} \approx 33.4,$$

and the direction is

$$\theta = \tan^{-1}\left(\frac{2.04}{-33.3}\right) + 180^\circ \approx -3.5^\circ + 180^\circ \approx 176.5^\circ,$$

or 176.5° counterclockwise from the positive x-axis. 180° is added because the resultant is in the second quadrant, and the arctangent function only gives values in the first and fourth quadrants.

Part (d)

The resultant is given as $\vec{\mathbf{R}} = 5.65\hat{\mathbf{i}} + 1.01\hat{\mathbf{j}}$. Its magnitude is

$$R = |\vec{\mathbf{R}}| = \sqrt{(5.65)^2 + (1.01)^2} \approx 5.74,$$

and the direction is

$$\theta = \tan^{-1}\left(\frac{1.01}{5.65}\right) \approx 10.1^\circ,$$

or 10.1° counterclockwise from the positive x-axis.

Part (e)

The resultant is

$$\begin{aligned}\vec{\mathbf{D}} - \vec{\mathbf{F}} &= (-20.0\cos 37^\circ\,\hat{\mathbf{i}} + 20.0\sin 37^\circ\,\hat{\mathbf{j}}) - (-20.0\cos 30^\circ\,\hat{\mathbf{i}} - 20.0\sin 30^\circ\,\hat{\mathbf{j}}) \\ &= (-20.0\cos 37^\circ + 20.0\cos 30^\circ)\hat{\mathbf{i}} + (20.0\sin 37^\circ + 20.0\sin 30^\circ)\hat{\mathbf{j}} \\ &\approx 1.35\hat{\mathbf{i}} + 22.0\hat{\mathbf{j}}.\end{aligned}$$

Its magnitude is

$$\left|\vec{\mathbf{D}} - \vec{\mathbf{F}}\right| \approx \sqrt{(1.35)^2 + (22.0)^2} \approx 22.1,$$

and the direction is

$$\theta = \tan^{-1}\left(\frac{22.0}{1.35}\right) \approx 86.5^\circ,$$

or 86.5° counterclockwise from the positive x-axis.

Part (f)

The resultant is

$$\begin{aligned}\vec{\mathbf{A}} + 2\vec{\mathbf{F}} &= (10.0\cos 30^\circ\,\hat{\mathbf{i}} + 10.0\sin 30^\circ\,\hat{\mathbf{j}}) + 2(-20.0\cos 30^\circ\,\hat{\mathbf{i}} - 20.0\sin 30^\circ\,\hat{\mathbf{j}})\\ &= (10.0\cos 30^\circ - 40.0\cos 30^\circ)\hat{\mathbf{i}} + (10.0\sin 30^\circ - 40.0\sin 30^\circ)\hat{\mathbf{j}}\\ &\approx -26.0\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}}.\end{aligned}$$

Its magnitude is

$$\left|\vec{\mathbf{A}} + 2\vec{\mathbf{F}}\right| = \sqrt{(-26.0)^2 + (-15.0)^2} = 30.0,$$

and the direction is

$$\theta = \tan^{-1}\left(\frac{-15.0}{-26.0}\right) + 180^\circ = 30^\circ + 180^\circ = 210^\circ,$$

or 210° counterclockwise from the positive x-axis (the same direction as $\vec{\mathbf{F}}$). 180° is added because the resultant is in the third quadrant, and the arctangent function only gives values in the first and fourth quadrants.

Part (g)

The resultant is

$$\begin{aligned}\vec{\mathbf{C}} - 2\vec{\mathbf{D}} + 3\vec{\mathbf{F}} &= (12.0\cos 60^\circ\,\hat{\mathbf{i}} - 12.0\sin 60^\circ\,\hat{\mathbf{j}}) - 2(-20.0\cos 37^\circ\,\hat{\mathbf{i}} + 20.0\sin 37^\circ\,\hat{\mathbf{j}})\\ &\quad + 3(-20.0\cos 30^\circ\,\hat{\mathbf{i}} - 20.0\sin 30^\circ\,\hat{\mathbf{j}})\\ &= (12.0\cos 60^\circ + 40.0\cos 37^\circ - 60.0\cos 30^\circ)\hat{\mathbf{i}} + (-12.0\sin 60^\circ - 40.0\sin 37^\circ - 60.0\sin 30^\circ)\hat{\mathbf{j}}\\ &\approx -14.0\hat{\mathbf{i}} - 64.5\hat{\mathbf{j}}.\end{aligned}$$

Its magnitude is

$$\left|\vec{\mathbf{C}} - 2\vec{\mathbf{D}} + 3\vec{\mathbf{F}}\right| = \sqrt{(-14.0)^2 + (-64.5)^2} \approx 66.0,$$

and the direction is

$$\theta = \tan^{-1}\left(\frac{-64.5}{-14.0}\right) + 180^\circ \approx 77.7^\circ + 180^\circ = 257.7^\circ,$$

or 257.7° counterclockwise from the positive x-axis. 180° is added because the resultant is in the third quadrant, and the arctangent function only gives values in the first and fourth quadrants.

Part (h)

The resultant is

$$\begin{aligned}\overrightarrow{\mathbf{A}} - 4\overrightarrow{\mathbf{D}} + 2\overrightarrow{\mathbf{F}} &= (10.0\cos 30^\circ\,\hat{\mathbf{i}} + 10.0\sin 30^\circ\,\hat{\mathbf{j}}) - 4(-20.0\cos 37^\circ\,\hat{\mathbf{i}} + 20.0\sin 37^\circ\,\hat{\mathbf{j}}) \\ &\qquad + 2(-20.0\cos 30^\circ\,\hat{\mathbf{i}} - 20.0\sin 30^\circ\,\hat{\mathbf{j}}) \\ &= (10.0\cos 30^\circ + 80.0\cos 37^\circ - 40.0\cos 30^\circ)\hat{\mathbf{i}} + (10.0\sin 30^\circ - 80.0\sin 37^\circ - 40.0\sin 30^\circ)\hat{\mathbf{j}} \\ &\approx 37.9\hat{\mathbf{i}} - 63.1\hat{\mathbf{j}}.\end{aligned}$$

Its magnitude is

$$\left|\overrightarrow{\mathbf{A}} - 4\overrightarrow{\mathbf{D}} + 2\overrightarrow{\mathbf{F}}\right| = \sqrt{(37.9)^2 + (-63.1)^2} \approx 73.7,$$

and the direction is

$$\theta = \tan^{-1}\left(\frac{-63.1}{37.9}\right) \approx -59.0^\circ,$$

or 59.0° clockwise from the positive x-axis.